

叶栅绕流计算的高分辨率隐式解法*

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【摘要】 以一种性能优越的高分辨率格式为基础, 发展了其隐式解法, 提出并实现了与这种格式相适应的隐式边界处理过程, 从而显著提高了高分辨率格式的收敛速度。用不同来流条件下叶栅绕流数组计算结果与实验做了对比, 以说明解的准确程度和本文算法的效果。

主题词: 高分辨率 叶栅流动 数值计算 隐式法

分类号: TB 115 O 241

1 引言

高分辨率差分格式在消除间断上下游的数值波动和提高对流场的分辨能力方面具有突出的优越性。但是, 高分辨率格式本身比较复杂, 运算工作量大, 采用显式方法获得一个定常解所需时间推进步数多、计算时间长。此外, 显式方法对于叶栅绕流的某些情况, 可能出现残差徘徊不降或下降极慢的问题。因此, 发展无条件稳定的隐式算法很有必要。但是, 在以往的隐式高分辨率差分格式的应用中, 边界条件的处理还大都采用显式方法, 这无疑又导入了稳定性限制, 从而影响到这种算法优势的充分发挥。不难推断, 发展内点和边界统一的全隐式算法, 将可能是克服这一障碍的最有效途径。本文以Yang^[1]发展的以特征通量差分裂为基础的高分辨率格式建立了隐式算法, 构造了特征隐式边界处理过程。实际计算表明, 这种算法不仅保持了解的无波动、高分辨率的特点, 而且具有很高的收敛速度, 所完成的跨音速压气机叶栅绕流计算, CFL 数可取 20 以上, 计算结果与实验吻合良好。

2 控制方程、高分辨率无波动隐格式

任意曲线坐标系下 (ξ, η) 的二维 Euler 方程为:

$$\partial Q / \partial \xi + \partial E / \partial \eta + \partial F / \partial \eta = 0 \quad (1)$$

其中 $Q = J^{-1}[\rho, \rho u, \rho v, e]^T$; $E = J^{-1}[\rho U, \rho u U + \xi p, \rho v U + \xi p, (e + p)U]^T$; $F = J^{-1}[\rho V, \rho u V + \eta p, \rho v V + \eta p, (e + p)V]^T$; $U = u\xi + v\eta$; $V = u\eta + v\xi$; $J = \partial(\xi, \eta) / \partial(x, y)$; ρ 为密度; u, v 分别为 x 和 y 向速度分量; e 为总内能; 压力 $p = (\gamma - 1)[e - \rho(u^2 + v^2)/2]$; γ 为比热比。

Jacobian 系数阵 $A = \partial E / \partial Q$ 和 $B = \partial F / \partial Q$ 具有实特征值 a_l 和 b_l , ($l = 1, 2, 3, 4$), 即:

$$a_1 = a_2 = U, \quad a_3 = U + c|\nabla \xi|, \quad a_4 = U - c|\nabla \xi|$$

和
$$b_1 = b_2 = V, \quad b_3 = V + c|\nabla \eta|, \quad b_4 = V - c|\nabla \eta|$$

其中 c 为音速。

Yang 发展的以通量差分裂为基础的 TVD 格式, 在对定常和非定常, 内流和外流场的数

值模拟中得到了十分出色的计算结果^[1,2]。采用时间向后积分,式(1)和隐格式为:

$$[\hat{I} + \Delta \hat{\mathcal{A}}_{j-1/2}^+ \Delta_- A_j + \Delta \hat{\mathcal{A}}_{j+1/2}^- \Delta_+ A_j + \Delta \hat{\mathcal{B}}_{k-1/2}^+ \Delta_- B_k + \Delta \hat{\mathcal{B}}_{k+1/2}^- \Delta_+ B_k] \hat{\mathcal{Q}}_{j,k}^{n+1} = \text{RHS}_{j,k} \quad (2)$$

其 AF-ADI 型式为

$$\left. \begin{aligned} [\hat{I} + \Delta \hat{\mathcal{A}}_{j-1/2}^+ \Delta_- A_j + \Delta \hat{\mathcal{A}}_{j+1/2}^- \Delta_+ A_j] \hat{\mathcal{Q}}_{j,k}^* &= \text{RHS}_{j,k} \\ [\hat{I} + \Delta \hat{\mathcal{B}}_{k-1/2}^+ \Delta_- B_k + \Delta \hat{\mathcal{B}}_{k+1/2}^- \Delta_+ B_k] \hat{\mathcal{Q}}_{j,k}^{n+1} &= \hat{\mathcal{Q}}_{j,k}^* \\ \hat{\mathcal{Q}}_{j,k}^{n+1} &= \hat{\mathcal{Q}}_{j,k}^* + \hat{\mathcal{Q}}_{j,k}^{n+1} \end{aligned} \right\} \quad (3)$$

其中 $\text{RHS}_{j,k} = -\Delta \tau \hat{\mathcal{A}}_{j-1/2}^+ \Delta_- E_j^M + \hat{\mathcal{A}}_{j+1/2}^- \Delta_+ E_j^M - \Delta \tau \hat{\mathcal{B}}_{k-1/2}^+ \Delta_- F_j^M + \hat{\mathcal{B}}_{k+1/2}^- \Delta_+ F_j^M$; $E_j^M = E_j + D_j$; D_j 的分量为 $d_j^l = \min \text{mod} [\hat{d}_{j+1/2}^l, \hat{d}_{j-1/2}^l]$; 而 $\hat{d}_{j+1/2}^l$ 是列向量 $\hat{D}_{j+1/2} = \text{sgn} A_{j+1/2} (I - \lambda |A|) \Delta_+ E_j / 2$ 的分量; $\lambda = \Delta t / \Delta \xi$; $A^\pm = L^{-1} \text{diag} \{a_i^\pm\} L$; $a_i^\pm = [1 \pm \text{sgn}(a_i)] / 2$; $\hat{A} = L^{-1} \text{diag} [\text{sgn}(a_i)]$; sgn 为符号函数; Δ_- 和 Δ_+ 分别为向后和向前差分算子。 F_j^M 可仿此写出。

3 隐式边界处理

为与内点隐格式相适应,从特征理论出发,发展了隐式边界过程。周期边界条件在计算格式的各项时直接应用。这里着重讨论进出口和物面边界。

以系数 Jacobian 阵的左特征阵 L 左乘(2)式:

$$L [\hat{I} + \Delta \tau (\hat{\mathcal{A}}^+ + \hat{\mathcal{A}}^-) \Delta A + \Delta \tau (\hat{\mathcal{B}}^+ + \hat{\mathcal{B}}^-) \Delta B] \hat{\mathcal{Q}}^{n+1} = L \cdot \text{RHS} \quad (4)$$

在边界上,(4)式中的 L 必须包含恰当的边界信息,为此,将(4)式改写为:

$$[\hat{L}_1 + \Delta \tau L_2 (\hat{\mathcal{A}}^+ + \hat{\mathcal{A}}^-) \Delta A + \Delta \tau L_2 (\hat{\mathcal{B}}^+ + \hat{\mathcal{B}}^-) \Delta B] \hat{\mathcal{Q}}^{n+1} = L_2 \cdot \text{RHS} \quad (5)$$

其中 L_2 中仅包含代表信息由计算区域向边界传播的左特征矢量,其它为零。

给定边界条件 $B_{ji} = 0$ (i 视具体边界而定),则有 $\partial B_i / \partial \tau = \partial B_i / \partial \eta = \partial B_i / \partial \alpha = 0$,将 $\partial B_i / \partial \eta$ 补在 L_2 中的零矢量处就得到 L_1 ,因此,(5)式的实质是边界上的特征相容关系与给定的边界条件联立构成的隐式边界求解系统。

对亚音速进口,给定来流的总压 p^* ,总温 T^* 和气流角 $\alpha = \tan^{-1}(v/u)$,则 $s = p / \rho^v$ 确定

$$L_1 = \begin{bmatrix} \partial s & \partial_u s & \partial_v s & \partial s \\ \partial T^* & \partial_u T^* & \partial_v T^* & \partial T^* \\ \partial \alpha & \partial_u \alpha & \partial_v \alpha & \partial \alpha \\ l_4 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ l_4 \end{bmatrix}$$

l_4 为 $U - c |\nabla \xi|$ 对应的左特征矢量。

对下物面,给定相切条件 $\eta_k \rho_u + \eta_l \rho_v = 0$

$$L_1 = \begin{bmatrix} l_1 \\ l_2 \\ 0 & \eta_k & \eta_l & 0 \\ l_4 \end{bmatrix}, \quad L_2 = \begin{bmatrix} l_1 \\ l_2 \\ 0 \\ l_4 \end{bmatrix}$$

l_1, l_2 和 l_4 分别为 $V, V, V - c |\nabla \eta|$ 对应的左特征矢量。其他边界的 L_1 和 L_2 可以类似地写出。

对(4)式的求解有两种方法,一是先左乘 L^{-1} ,然后进行 AF 分解^[3]。另一种方法^[4]是先进 AF 分解,然后左乘 L_1 和 L_2 。虽然后一种方法的计算量较前一种方法小些,但并不严格,文献[4]证明了这样引入的误差和时间一阶 Beam-Warming 格式的截断误差为同一量级。

在叶栅绕流计算中,比较了这两种做法,结果没有明显区别。本文结果中,除 GAMM 算例采用后一种做法外,其余均采用第一种做法。

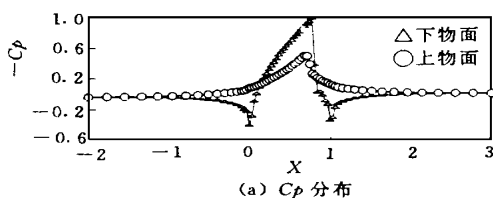
4 算例分析

算例包括 GAMM 流道亚音和超音进口两种情况以验证程序, 另外 3 个算例是高亚音、微弱超音和较强超音进口叶栅流动。采用的收敛准则是残差

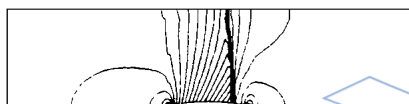
$$R = \left\{ \sum_{j=1}^{J_{\max}} \sum_{k=1}^{K_{\max}} \left[\left| \rho_{j,k}^{n+1} - \rho_{j,k}^n \right| / (\Delta \tau \cdot \rho_{j,k}^n) \right] \right\} / (J_{\max} \cdot K_{\max}) \leq 10^{-3}$$

由于 $\Delta \tau$ 很小, 因此这一标准是严格的。

GAMM 流道。下物面圆弧的弦长为 1, 最大相对厚度 4.2%, 流道高为 1, 来流 Ma 数 0.8。图 1(a) 和 (b) 分别给出了上下物面的 C_p 沿流向分布和等 Ma 数图。在约 78% 处获得一道激波, 占两个网格宽。上下物面的 C_p 在进出口附近重合为零。计算取 CFL = 25, 推进 52 步收敛, 采用显式边界处理, CFL 数只能取至 7^[1]。



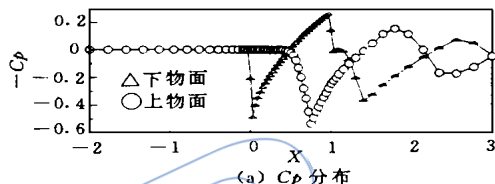
(a) C_p 分布



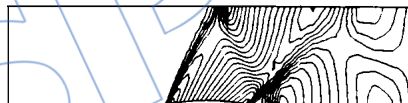
(b) 等 Ma 数分布

图 1 GAMM 流道亚音进口计算结果

来流 Ma 数 1.4。图 2(a) 和 (b) 分别给出了上下物面的 C_p 分布和等 Ma 数图, CFL 数取 20, 推进 40 步收敛。捕捉到的前缘和后缘激波均占两个网格宽。前缘激波在上物面形成反射波和后缘激波相交。计算给出的波系清晰, C_p 分布合理。图 3 是下物面熵增分布, 两道激波均产生陡峭熵增, 光滑区域熵水平维持不变。一般方法的计算很难给出如此规则的熵增分布。



(a) C_p 分布



(b) 等 Ma 数分布

图 2 GAMM 流道超音进口计算结果

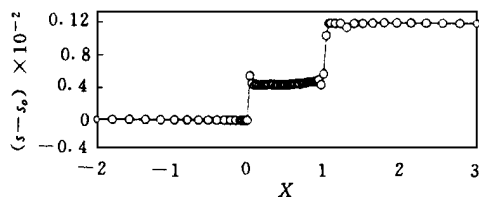


图 3 GAMM 流道超音进口熵增分布

CDA 41 叶栅。图 4 给出了进口 Ma 数 0.92 时压力面和吸力面 Ma 数分布和流场等 Ma 数图。除压力面前缘计算有些偏差外, 其余与实验吻合相当好。图 5 为该例的收敛史比较, CFL = 0.9 为显格式, 可见收敛极慢; CFL 取 25 时, 推进 77 步收敛。图 6 给出了进口 Ma 数 1.01 时的计算结果, 表面 Ma 数除压力面前缘局部区域外, 与实验亦基本吻合。

超音叶栅。图 7 为进口 Ma 数 1.2 的压气机叶栅 Ma 数分布和流场等 Ma 数图。计算显示, 叶栅被一较强激波封口, 气流穿过激波后随吸力面的曲率变化又发生少许膨胀。前缘已呈现外伸波的基本特征。

结论 本文建立的隐式解法能够准确快捷地计算具有复杂波系的叶栅流场。提出的特征隐式边界处理方法显著提高了高分辨格式的计算效率, 充分发挥了隐式格式的潜力, 且保持了

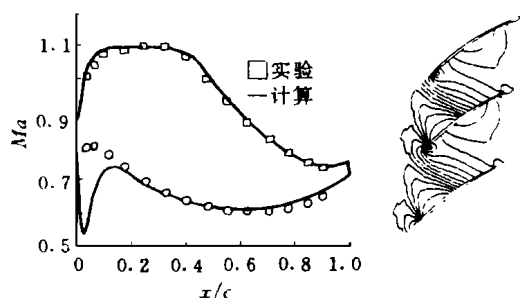


图4 CDA 叶栅亚音来流计算结果

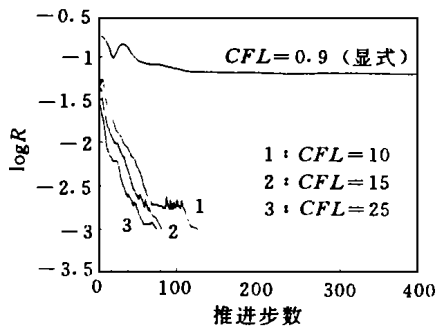


图5 CDA 叶栅亚音来流收敛史

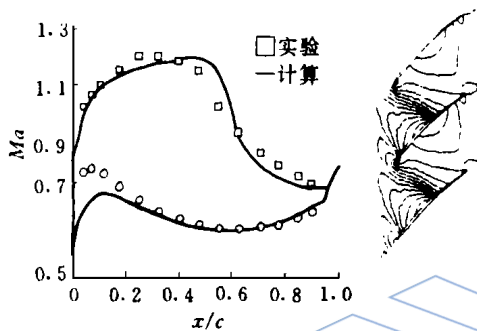


图6 CDA 叶栅超音来流计算结果

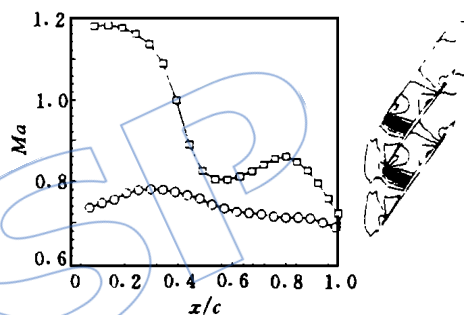


图7 超音叶栅计算结果

格式原有的分辨率, 从而为叶栅流动计算提供了一种准确、实用的手段。

参 考 文 献

- 1 Yang T Y. Third- Order Nonoscillatory Schemes for the Euler Equations A IAA J. , 1991, 29(10)
- 2 Yang T Y. High- Resolution, Nonoscillatory Schemes for Unsteady Compressible Flows A IAA J. , 1992, 30(6)
- 3 Chakravarthy S R. Euler Equations- Implicit Schemes and Boundary Conditions A IAA J. , 1983, 21(5)
- 4 Rai M M. New Implicit Boundary Procedures: Theory and Applications A IAA P. , 83-0123

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count, such as the collective pitch, precone, sweep, and so on. Finally, the aeroelastic stability of Dolphin's Rotor blades is analysed. The experiment shows that the aeroelastic stability of the rotor blade in hover is influenced small by the blade sweep, but strongly by the collective pitch and precone. The experimental results verify the feasibility of the analysis and applicability to rotor design.

Key words Helicopter rotor Hovering Aeroelastic dynamics Stability

A BLADE FLUTTER PREDICTION METHOD INCLUDING SECONDARY FLOW EFFECTS

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ABSTRACT In order to obtain a better understanding of the secondary flow effects on blade stall flutter, this paper presents an approximate method to analyse the effects of secondary flow on blade aeroelastic stability. The method combines conventional streamline curvature methods with the secondary flow model to obtain the stationary flow with circumferential nonuniformity, and then incorporates the non-constant phase angle deforming actuator disk method. Several secondary flow effects on aeroelastic stability were analysed respectively. The results show that the effects of the secondary flow on the aeroelastic stability are different. The tip leakage vortex may have greater effect than other vortices, that is consistent with the experimental results.

Key words Secondary flow Blade Aeroelastic Stability Predictions

NUMERICAL ANALYSIS OF FLOW FIELD ABOUT SWEEP-FORWARD BLADES OF COMPRESSORS OR FANS

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ABSTRACT A numerical analysis of the flow field about sweep-forward blades has been carried out. Time marching method is used to calculate the three-dimensional inviscid flow field. Boundary layer parameters are calculated with integrated boundary layer equations in non-orthogonal axes. The calculated results show that in axial compressors and fans with sweep-forward blades the intensity of passage vortex, secondary flow and tip losses are decreased, the early separation from the blade surface is protected, the flow field in tip area is improved obviously. As a result, the aerodynamic efficiency of axial compressor or ventilator is enhanced.

Key words Sweep-forward blades boundary layer Numerical analysis

HIGH-RESOLUTION IMPLICIT PROCEDURES FOR CASCADE FLOW CALCULATIONS

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ABSTRACT Based on the high-resolution difference scheme structured from characteristic flux difference splitting, the implicit algorithm of the scheme has been developed. The implicit characteristic boundary procedures consistent with the scheme have been implemented as well. The implicit boundary procedures proposed in this paper eliminate the

restrictions induced by explicit boundary treatments and, meanwhile, preserve all the important features of the scheme such as non-oscillation and high resolution. The implicit solver effectively accelerates the convergence rate of the high-resolution scheme. The comparison of cascade flow calculations under various inflow conditions with the corresponding experiments demonstrates the accuracy and efficiency of the proposed solver.

Key words Cascade flow Numerical computation High resolution Implicit methods

NEAR SOUND FIELD ANALYSIS OF A ROTATING SOURCE

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ABSTRACT On the basis of moving source radiation theory, this paper discusses the sound solution of a rotating source in frequency domain. Emphasis is laid on near sound field which has three "additional sources" in contrast to far field. The computation reveals that the near sound field produced by these "additional sources" distributes in discrete viewing angles. Radial and axial deviations affect sound field respectively around the rotating plane and axial area, and their maximum connections have different characteristics. The analysis is proved preliminarily by a fan experiment.

Key words Sound Radiation Aeroacoustics Sound source Frequency analysis

NUMERICAL ANALYSIS OF FLOW AND LEAKAGE IN LABYRINTH SEAL

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(National Lab. of Coal Combustion Huazhong University of Science & Technology, Wuhan, 430074)

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(Mechanical College, Nanjing University of Science & Technology)

ABSTRACT The unsteady Navier-Stokes equations are solved to predict numerically the internal flow of single-cavity labyrinth channel with depth-to-width ratios $H/W = 0.1, 0.3$ and 0.5 . The numerical results demonstrate the oscillation of the flow structure and fluctuation of the exit leakage rate. It is shown that the seal efficiency is relevant to the amplitude of leakage fluctuation, furthermore, the sequence of H/W from $0.3, 0.5$ to 0.1 is also the order of both increasing seal efficiency and increasing amplitude of leakage fluctuation. Also, the complex physics, including motion and mergences of vortex, migration of separation bubble, bending and impact of jet, is discussed, and the influence of impact effects on the increase of energy dissipation and decrease of leakage are analysed.

Key words Labyrinth seal oscillation flow Numerical analysis

METHOD FOR PREDICTING FATIGUE RELIABILITY AND LIFE OF COMPRESSOR BLADE

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ABSTRACT In engineering application of fatigue problem, it is very difficult to obtain some working parts' load spectrum due to some reasons. This paper provides method for predicting the fatigue reliability and life of the part without working load spectrum. Fatigue tests are carried out on the part that has worked for some time. Basing on the test data, the surplus fatigue damage strength of the part can be calculated. With its surplus fatigue